Lecture 16: Testing for equality of variances: the F-test

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- Today: Testing for equality of variances with an F test.

Measures of variability

Testing for equality of variances: the F-test

t - test for diffs in μ

z - test " " "

F - test for diff in σ^2

Measures of variability

- Variance $= Var(x) = \sigma_x^2$

Measures of variability

- Variance $= Var(x) = \sigma_x^2$
- Standard deviation $= \sqrt{Var(x)} = \sigma_x$

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The variance of a continuous probability distribution is

$$Var(x) = E(x-\mu)^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx. \quad = \sigma^2$$

Calculation of the sample variance

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Calculation of the sample variance

estimate of σ^2

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Sample standard deviation

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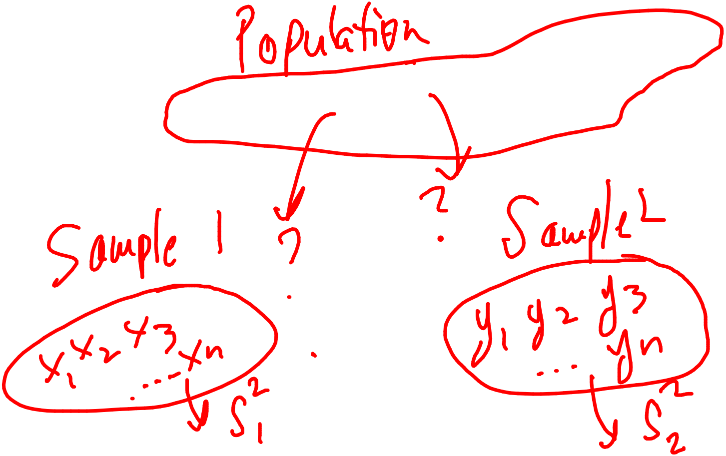
$n-1$ is called v, or the "degrees of freedom" used in the calculation of s.

F-test steps

1. We want to know if the two population variances are equal. Null hypothesis is that $\sigma_1^2 = \sigma_2^2$.



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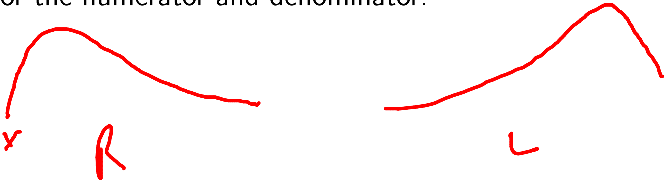
3. Form the ratio of the larger over the smaller.
4. This ratio has an F-distribution with degrees of freedom for the numerator and denominator.

$\frac{s_1^2}{s_2^2}$



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5. Compare the observed F with F_{crit}, which is qf(1 - alpha/2, dfn, dfd). Note that we divide alpha by 2 for a two-tailed test.

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4. This ratio has an F-distribution with degrees of freedom for the numerator and denominator.
5. Compare the observed F with F_{crit}, which is qf(1 - alpha/2, dfn, dfd). Note that we divide alpha by 2 for a two-tailed test.
6. Calculate the probability of the observed F as 2(1 - pf(F, dfn, dfd)). Note that we multiply by 2 if this is a two-tailed test.

Bumpus data

$$F = \frac{0.434}{0.176} = 2.47$$

$$p = 0.04$$

In females, the variance of the humerus length of survivors was 0.176, and the variance for the non-survivors was 0.434. The number of survivors was 21, and the number of non-survivors was 28. Question: is the population variance of survivors different from the population variance of non-survivors?

$\therefore$ reject H_0. Conclude

$$\sigma_s^2 \neq \sigma_{ns}^2$$

$$\sigma_{ns}^2 > \sigma_s^2$$

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Exam scores, men and women

In our first midterm exam, the sample variance for women was 59.2, and the sample variance for men was 68.9. There were 14 women and 13 men taking the exam. Is the true population variance for women different from that for men?

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Low end of 95% confidence interval:

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High end of 95% confidence interval: $\frac{vs^2}{\chi^2_{0.025}}$

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